# Cyclostationary Empirical Orthogonal Function (CSEOF) Reconstructed Sea Level User's Handbook

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#### 1. DATA PRODUCT ABSTRACT

This product contains near-global reconstructed sea level fields from 1950 to 2010. The reconstructed fields are obtained by fitting satellite altimetry-derived basis functions, computed using cyclostationary empirical orthogonal functions (CSEOF), to historical tide gauge data. The result is a dataset with the spatial resolution of the satellite altimetry and the record length of the tide gauges.

### 2. INVESTIGATOR'S NAME AND CREDIT

The CSEOF reconstructed sea level dataset was created at the University of Colorado at Boulder by Benjamin Hamlington, Robert Leben and Kwang-Yul Kim.

If using the reconstructed data for publications, please include the following citation:

The CSEOF reconstructed sea level data was obtained from JPL Physical Oceanography DAAC and developed by the University of Colorado.

The following reference can also be cited:

Hamlington B.D., Leben R.R., Nerem R.S., Han, W., Kim K.-Y., 2011: Reconstructing sea level using cyclostationary empirical orthogonal functions, *Jour. Geophys. Res*, 116, C12015.

### 3. DATA DESCRIPTION

#### 3.1 File Format and Naming

The file format for the data is a single NetCDF file containing all reconstructed sea level fields from 1950 to 2010. The file is named by the following convention:

CCAR recon sea level YYYYMMDD YYYYMMDD Vn.nc

Where YYYYMMDD is the start (first) or end (last) date of the data contained within the file. Vn is the version number of the file, for example V1 will be version 1.

### 3.2 Temporal Coverage and Resolution

The reconstructed sea level fields extend from June of 1950 to the middle of 2010 and will be updated periodically. As a result of the one-year windowing of the CSEOF reconstruction procedure, half a year of data is lost at each end of the reconstruction. The data has weekly resolution, and is provided in the NetCDF file has days since 1/1/1900.

## 3.3 Spatial Coverage and Resolution

The latitudes of 90°S to 90°N are provided, although the reconstructed data fields extend only to 65°S to 65°N with anything north or south of that extent given fill values in the sea surface height anomaly variable. The data covers longitudes from  $0^{\circ}$  to  $360^{\circ}$ . The resolution is  $0.5^{\circ}$  in each direction.

### 3.4 Data Parameters and Format

### **LONGITUDE**

Parameter	Value
Long Name	Longitude
Standard Name	Longitude
Units	Degrees East
Dimension	720
Valid Range	[0 360]
Maximum	360
Minimum	0
Spacing	0.5

### LATITUDE

Parameter	Value
Long Name	Latitude
Standard Name	Latitude
Units	Degrees North
Dimension	361
Valid Range	[-90 90]
Maximum	90
Minimum	-90
Spacing	0.5

### TIME

Parameter	Value
Long Name	Time (days since 1/1/1900)
Standard Name	Time
Units	Days since 1/1/1900
Dimension	3094
Spacing	7 days

#### SEA SURFACE HEIGHT ANOMALY

Parameter	Value
Long Name	Reconstructed SSHA
Standard Name	Sea_surface_height
Units	cm
Dimension	361 x 720 x 3094
Valid Range	[-200 200]
Fill Value	10^100
Scale Factor	0.1

### 4. PROCESSING METHODS

Using basis functions computed from a short, spatially dense data set to interpolate a long time series of spatially sparse observations was first implemented in sea surface temperature (SST) studies. Smith et al. [1996] computed empirical orthogonal functions (EOFs) from 12 years of satellite-derived SST data and used them as basis functions to estimate global SST temperature fields from 1950 to 1992. Kaplan et al. [1998, 2000] improved on this procedure by adding weighting dependent on known errors in the data to the reconstruction procedure. Sea level reconstructions soon followed using the techniques developed for SSTs. The mathematical details are found in the works by Kaplan et al. [2000], Church et al. [2004] and thus are not repeated in detail here.

The goal of reconstructing sea level is to create a sea level dataset with the record length of the tide gauge data and the spatial resolution of the satellite altimetry data. The central historical sea level dataset we use for the period 1950 through 2010 is monthly mean sea level records gathered from the data archive at the Permanent Service for Mean Sea Level (PSMSL). We use only the Revised Local Reference (RLR) data, which are measured sea level at each site relative to a constant local datum over the complete record. At present, we have not included the metric data offered by PSMSL as they can have substantial unknown datum shifts and their use in time series analysis is generally not recommended. The editing criteria implemented for the tide gauges is described in detail in Hamlington et al. [2011a].

The most significant difference in the creation of this sea level reconstruction compared to previous reconstructions is the use of cyclostationary empirical orthogonal functions (CSEOFs) in the place of EOFs as the basis for the reconstruction. Previous reconstructions – both SST and sea level – have generally relied on EOFs to form the basis for the reconstruction. When compared to CSEOFs, however, EOFs have characteristics that make them suboptimal for use as basis functions for sea level reconstruction. EOFs enforce a stationarity on the spatial variability. A single spatial map defines the basis function, and the reconstruction procedure simply computes the amplitude modulation of this map through time. Given the evidence that many signals in geophysical data are cyclostationary, CSEOFs provide significant advantages over EOFs when dealing with signals such as modulated annual cycle and ENSO signals. Although EOFs are able to represent cyclostationary features through a superposition of multiple modes, CSEOFs are able to explain cyclostationary signals in a single mode, increasing the opportunity for interpretability.

The decomposition of data in terms of a set of basis functions is often very useful in understanding the complicated response of a physical system. By decomposing into less complicated patterns, it may be easier to understand and shed light into the nature of the variability in a dataset. While theoretical basis functions have been studied extensively, exact theoretical basis functions are very difficult to find and in general, computational basis functions are sought instead. Perhaps the simplest and most common computational basis functions are EOFs. Consider a simple system defined by:

$$T(r,t) = \sum_{i} LV_{i}(r)PC_{i}(t)$$
 (1)

where LV(r) is a physical process (termed to be the loading vector) modulated by a stochastic time series PC(t), which is called the principal component time series. Each loading vector and principal component time series pair represents a single EOF mode. As mentioned above, however, physical processes and the corresponding statistics are time-dependent. Representing the data with stationary EOFs can lead to erroneous and difficult interpretation of the data [Dommenget and Latif, 2002].

Kim et al. [1996; 1997; 1999; 2001] introduced the concept of cyclostationary empirical orthogonal function (CSEOF) analysis to more compactly capture the time-varying spatial patterns and longer-timescale fluctuations present in geophysical signals. The significant difference between CSEOF and EOF analysis is the LVs' time dependence, which allows the spatial pattern of each CSEOF mode to vary in time, with the temporal evolution of the spatial pattern of the CSEOF LVs constrained to be periodic with a selected "nested period". In other words, the system is defined as:

$$T(r,t) = \sum_{i} LV_{i}(r,t)PC_{i}(t)$$

$$LV(r,t) = LV(r,t+d)$$
(2)

where the loading vectors are now time dependent, and are periodic with the nested period, d. As a result, each CSEOF mode is composed of twelve LVs and one PCTS when, for example, using monthly data and a one-year nested period. CSEOF analysis minimizes mode mixing, which is a common problem with EOF decomposition. When mode mixing occurs, the annual cycle frequently spreads across several modes, which is one reason the signal is usually removed from the data by some other means. Recent studies, however, have demonstrated the efficacy of using CSEOFs to extract robust modes representing the modulated annual cycle (MAC) and ENSO variability Hamlington et al., 2011a; 2011b]. This leads to robust estimates of the MAC or ENSO variability from satellite altimetry data without affecting signals associated with other ocean variability. For further details on CSEOFs and the procedure for computing CSEOFs, the reader should refer to Kim et al. [1997] in which a detailed description of the computation of CSEOFs is provided.

By fitting CSEOFs in place of EOFs to tide gauges to reconstruct sea level for the 1950 to 2010 time period [Hamlington et al., 2011b], we have found that it is possible to create an alternate and, we believe, improved reconstruction to those based on EOFs. Our motivation for using CSEOFs, as described in Hamlington et al. [2011b], in place of

EOFs is fourfold: 1) EOFs are not an optimal basis for non-stationary signals with nested oscillations that are undergoing low frequency oscillation; 2) CSEOFs account for both the high and low frequency components of the annual cycle and do not require the removal of the annual signal from the satellite altimetry nor the tide gauge records before reconstruction; 3) specific signals, such as those relating to the MAC and ENSO can be reconstructed individually using CSEOFs with little mixing of variability between modes; and 4) the reconstruction procedure using CSEOFs spans data gaps smaller than the nested period and fits a larger window of data, allowing for the use of fewer historical data to obtain meaningful results. In Hamlington et al., [2011b] the impact of tide gauge selection, editing, and weighting on the fidelity of the reconstructions was also investigated. Error analyses were performed using Monte Carlo simulations.

CSEOF sea level basis functions for our reconstruction were estimated from the AVISO quarter-degree resolution multiple altimeter product based on satellite altimeter measurements spanning 1993-2011 collected by the Topex/Poseidon, ERS-1&2, Geosat Follow-On, Envisat, Jason-1 and OSTM satellites. A nested period of one year was used in the CSEOF decomposition. We applied very little additional processing other than removing the mean and a linear least-square fit from the time series at each spatial grid point. A CSEOF decomposition of the satellite altimetry data is not able to extract the change in mean sea level into a single mode. It is therefore necessary to remove mean sea level from the satellite altimetry data before computing the basis functions to avoid putting low-frequency power into each CSEOF mode. The global mean sea level component of the reconstruction is introduced after the reconstruction of the basis functions is complete.

For further details on the procedure and details therein, users are encouraged to read Hamlington et al. [2011b], which describes a reconstructed sea level dataset very similar to the one provided here.

#### 5. SOURCES OF ERROR

We do not provide an estimate of the errors in the reconstruction sea level fields. Errors can be estimated as a combination of instrument errors and the error resulting from truncating the basis functions estimated from the AVISO dataset. There is also error related to the poor historical distribution of the tide gauges back through time. A discussion of error and general overview of the level of the error is provided in Hamlington et al. [2011b], and as a future update, and more comprehensive treatment of the errors in the reconstruction may be included as part of this dataset. For further details, the user is referred to Hamlington et al. [2011b], where an estimate of the error on the global mean sea level from this dataset is provided.

#### 6. REFERENCES

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### 7. ACRONYMS

CSEOF – Cyclostationary Empirical Orthogonal Functions

EOF – Empirical Orthogonal Functions

GMSL – Global Mean Sea Level

SSH – Sea Surface Height

SST – Sea Surface Temperature

### 8. CONTACT INFORMATION

Questions or comments about this data product should be directed via email to the Physical Oceanography DAAC: podaac@podaac.jpl.nasa.gov.

### 9. DOCUMENT INFORMATION

Benjamin Hamlington, University of Colorado Jessica Hausman, Jet Propulsion Laboratory Robert Leben, University of Colorado Michael Shannon, University of Colorado February, 2013

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